# Relation between Stress-Strain Behavior and Equilibrium Volume Swelling for Peroxide Vulcanizates of Natural Rubber and cis-1,4-Polyisoprene 

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## Synopsis

Data are reported for the elastic behavior, as described by the parameters $C_{1}$ and $C_{2}$, and the equilibrium swelling in $n$-decane, $v_{r}$, for dicumyl peroxide vulcanizates of natural rubber and cis-1,4-polyisoprene. For natural rubber vulcanizates the correlation between $C_{1}$ and $v_{r}$ can be described in terms of the original Flory-Rehner equation with $x=0.41$. Tentative evidence is presented which favors this equation rather than the later modified form in which the term in $v_{r}{ }^{1 / 2}$ is replaced by one in $\left(v_{r}^{1 / 2}-v_{r} / 2\right)$.

In this note data are presented which define the correlation between the equilibrium volume swelling in $n$-decane of dicumyl peroxide vulcanizate networks of natural rubber and synthetic cis-1,4-polyisoprene and their stress-strain behavior in simple extension as interpreted in terms of the Mooney-Rivlin equation; ${ }^{1,2}$

$$
\begin{equation*}
f=2 A_{0}\left(\lambda-\lambda^{-2}\right)\left(C_{1}+\lambda^{-1} C_{2}\right) \tag{1}
\end{equation*}
$$

In this equation $C_{1}$ and $C_{2}$ are constants characterizing the vulcanizate network, and $f$ is the force required to extend the vulcanizate network, initially of cross-sectional area $A_{0}$, to an extension ratio $\lambda$. The stressstrain behavior of swollen vulcanizates can be similarly interpreted; $C_{1}$ is found to be nearly identical with that observed for the unswollen material, but $C_{2}$ is progressively reduced as the degree of swelling is increased. Ultimately for a highly swollen vulcanizate, if measurements can be made under conditions free from interference from finite chain extensibility effects, $C_{2} \rightarrow 0$ and,

$$
\begin{equation*}
f=2 A_{0}\left(\lambda-\lambda^{-2}\right) C_{1} \tag{2}
\end{equation*}
$$

$2 C_{1}$ is therefore to be identified with the constant, $G$, of the equation derived from statistical theory, ${ }^{3}$

$$
\begin{equation*}
\underset{1571}{f / A_{0}=} \underset{ }{G\left(\lambda-\lambda^{-2}\right)} \tag{3}
\end{equation*}
$$

TABLE I
Measurements of $C_{1}, C_{2}$, and $v_{r}$ at $25^{\circ} \mathrm{C}$.

|  | $\begin{gathered} {[\eta]} \\ \mathrm{d} 1 . / \mathrm{g} . \end{gathered}$ | $v_{r}$ | $\begin{gathered} C_{1} \\ \text { lyne/cm. }{ }^{2} \end{gathered}$ | $\begin{gathered} C_{2}, \\ \text { lyne/cm. }{ }^{2} \end{gathered}$ | Sol rubber, | $\begin{aligned} & C_{1} \text { (corr.) } \\ & \text { dyne } / \mathrm{cm} .^{2} \end{aligned}$ |  |  |  | Benzene |  |  | Heptan |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ( toluene) | -decane) | $\times 10^{-6}$ | $\times 10^{-6}$ | wt.-\% | $\times 10^{-6}$ | $\chi_{0}$ | $\chi^{\prime} \mathrm{D}$ | $v_{\text {r }}$ | $\chi^{\prime}{ }_{B}$ | $\chi^{\prime}$ в | $v_{r}$ | $\chi_{\mathbf{H}}$ | $\chi^{\prime}$ н |
| Natural rubber vulcan- | 2.13 | 0.0641 | 0.118 | 0.563 | 18.7 | 0.136 | 0.314 | 0.329 |  |  |  |  |  |  |
| ized with dicumyl | " | 0.0947 | 0.156 | 0.441 | 11.3 | 0.169 | 0.398 | 0.411 |  |  |  |  |  |  |
| peroxide at 140- | " | 0.1069 | 0.242 | 0.617 | 9.9 | 0.259 | 0.368 | 0.385 |  |  |  |  |  |  |
| $150{ }^{\circ} \mathrm{C}$. | " | 0.1218 | 0.331 | 0.637 | 8.2 | 0.350 | 0.366 | 0.387 |  |  |  |  |  |  |
|  | " | 0.1401 | 0.381 | 0.640 | 6.5 | 0.398 | 0.385 | 0.406 | 0.1021 | 0.416 | 0.429 | 0.1470 | 0.449 | 0.463 |
|  | " | 0.1562 | 0.462 | 0.667 | 5.1 | 0.478 | 0.384 | 0.406 |  |  |  |  |  |  |
|  | " | 0.1788 | 0.534 | 0.643 | 3.1 | 0.544 | 0.394 | 0.419 |  |  |  |  |  |  |
|  | ' | 0.1954 | 0.646 | 0.725 | 2.1 | 0.655 | 0.418 | 0.449 | 0.1426 | 0.428 | 0.445 | 0.2047 | 0.470 | 0.489 |
|  | " | 0.2055 | 0.749 | 0.760 | 1.9 | 0.758 | 0.414 | 0.443 |  |  |  |  |  |  |
|  | 2.12 | 0.2228 | 0.922 | 0.750 | 1.9 | 0.933 | 0.409 | 0.441 |  |  |  |  |  |  |
|  | 2.13 | 0.2408 | 1.072 | 0.802 |  |  | 0.416 | 0.451 |  |  |  |  |  |  |
|  | 3.02 | 0.2494 | 1.110 | 0.644 |  |  | 0.425 | 0.460 |  |  |  |  |  |  |
|  | 2.14 | 0.2730 | 1.429 | 0.794 |  |  | 0.418 | 0.459 | 0.1993 | 0.428 | 0.453 | 0.2865 | 0.486 | 0.515 |
|  | 2.16 | 0.2850 | 1.690 | 0.899 |  |  | 0.404 | 0.451 |  |  |  |  |  |  |
|  | " | 0.3024 | 1.842 | 0.880 |  |  | 0.416 | 0.464 |  |  |  |  |  |  |
|  | - | 0.3056 | 1.846 | 0.864 |  |  | 0.422 | 0.470 |  |  |  |  |  |  |
|  | 3.19 | 0.3147 | 2.113 | 0.741 |  |  | 0.408 | 0.461 |  |  |  |  |  |  |
|  | 2.22 | 0.3163 | 2.076 | 0.860 |  |  | 0.415 | 0.467 | 0.2309 | 0.420 | 0.452 | 0.3300 | 0.490 | 0.528 |
|  | - | 0.3290 | 2.310 | 0.884 |  |  | 0.415 | 0.470 |  |  |  |  |  |  |
|  | 2.20 | 0.3322 | 2.356 | 0.940 |  |  | 0.415 | 0.471 |  |  |  |  |  |  |
|  | - | 0.3459 | 2.706 | 0.864 |  |  | 0.407 | 0.468 |  |  |  |  |  |  |
|  | - | 0.3480 | 2.734 | 0.896 |  |  | 0.407 | 0.469 |  |  |  |  |  |  |
|  | - | 0.3512 | 2.672 | 0.846 |  |  | 0.418 | 0.480 |  |  |  |  |  |  |
|  | - | 0.3560 | 2.894 | 0.817 |  |  | 0.407 | 0.472 |  |  |  |  |  |  |
|  | 3.04 | 0.3559 | 2.770 | 0.880 |  |  | 0.419 | 0.480 |  |  |  |  |  |  |


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${ }^{a} C_{1}$ corrected as detailed in the text for presence of sol rubber.
 cis-1,4-Polyisoprene
vulcanized with di-
cumyl peroxide at
$140-150^{\circ} \mathrm{C}$.
and is a measure of the degree of physically effective crosslinking. $C_{2}$ is clearly a term characterizing departures fom this ideal behavior, but its actual physical significance is not apparent at present.

The measurements reported here differ in two respects from similar data for natural rubber published by Mullins. ${ }^{4}$ Firstly, the range of $C_{1}$ values has been extended from $0.4<C_{1}<2.0$ to $0.2<C_{1}<3.8$ dyne $/ \mathrm{cm} .{ }^{2}{ }^{2} \times$ $10^{6}$. Secondly, $C_{1}$ has been measured by means of the apparatus and technique described by Greensmith. ${ }^{5}$ Greensmith's technique involves the determination of stress for a predetermined series of $\lambda$ values, the sample undergoing some measure of stress relaxation, whereas the method used by Mullins ${ }^{4}$ consisted of determining $\lambda$ for a given stress level and


Fig. 1. Relation between the elastic constant $C_{1}$ and the equilibrium volume swelling $v_{r}$.
involved therefore a certain amount of creep. The present data give further confirmation of the identity of the estimates of $C_{1}$ made by these two methods.

Acetone-extracted pale crepe natural rubber and synthetic ca. $92 \%$ cis-1,4-polyisoprene (Cariflex 305, Shell Chemical Co.) were each mixed on the mill with $1-5 \mathrm{phr}$ of recrystallized dicumyl peroxide and vulcanized as 1 mm . sheets at $140-150^{\circ} \mathrm{C}$. The vulcanized sheets were extracted with hot acetone for 24 hr . and dried in vacuo; this process removed residual peroxide and peroxide decomposition products but not any rubber remaining unvulcanized (sol rubber). Equilibrium volume swelling in $n$-decane at $25^{\circ} \mathrm{C}$., characterized as $v_{r}$, the volume fraction of rubber network in the swollen gel, was determined as described previously. ${ }^{4}$ For all but the most lightly crosslinked vulcanizates, the quantity of soluble material extracted during the $n$-decane swelling measurement was negligible, and hence the
$v_{r}$ values estimated referred to a vulcanizate network which had received a comparable extraction treatment and undergone therefore a similar extent of supercoiling to that used for the $C_{1}$ determination. For the few very lightly vulcanized materials, from which an appreciable amount of sol rubber was extracted during swelling in $n$-decane, this conclusion as regards supercoiling is no longer valid. A correction was applied therefore to the cross-sectional area of the vulcanizate used in the $C_{1}$ determination, $C_{1}$ being increased by the factor $[100 /(100-\text { volume- } \% \text { sol rubber })]^{2 / 8}$. This correction, which is the same as that used by Mullins, ${ }^{4}$ will be discussed in detail elsewhere. ${ }^{6}$


Fig. 2. Relation between the constants $C_{1}$ and $C_{2}$. ( $C_{1}$ and $C_{2}$ are expressed in the same units.)
$C_{1}, C_{2}$, and $v_{r}$ data are given in Table I. The plot of $C_{1}$ versus $v_{r}$ (Fig. 1) is, for natural rubber, identical over the relevant $C_{1}$ range with that observed by Mullins. ${ }^{4}$ cis-1,4-Polyisoprene vulcanizates seem to conform to a slightly different $C_{1}-v_{r}$ relation to that found for natural rubber. In Figure 2 the parameter $C_{2}$ is plotted as a function of $C_{1}$; within the rather large scatter, $C_{2}$ appears to be a smooth function of $C_{1}$ with a maximum at $C_{1}=$ ca. 2.0. Slightly higher $C_{2}$ values are observed for cis-1,4-polyisoprene.

The theoretical relation between $C_{1}$ and $v_{r}$ in the form originally proposed by Flory ${ }^{7}$ is,

$$
\begin{equation*}
-\ln \left(1-v_{r}\right)-v_{r}-\chi_{\mathrm{D}} v_{r}^{2}=\left(2 V_{0} C_{1} / R T\right) v_{r}^{1 / 2} \tag{4}
\end{equation*}
$$

where $V_{0}$ is the molar volume of the solvent ( $n$-decane) and $\chi_{0}$ the solventpolymer interaction parameter. Values of $\chi_{D}$ obtained by application of
this equation are included in Table I. A value of $\chi_{D}=c a .0 .41$ is indicated over most of the $v_{r}$ range. For data for which $v_{r}>0.18$ the arithmetic mean value of $\chi_{D}$ is 0.411 (standard deviation 0.008 ) and the line in Figure 1 is drawn for this value.
$\chi_{\mathrm{D}}$ for cis-1,4-polyisoprene vulcanizate networks is slightly larger than for natural rubber networks, especially at low $v_{\tau}$ values, though such a difference is not observed ${ }^{8}$ in the osmotic behavior of the two rubbers in dilute solution ( $v_{r}<0.015$ ) in toluene. While this difference in $\chi_{\mathrm{D}}$ may be real, it could be an artifact arising in the estimation of $C_{1}$ from the plots of $f\left[2 A_{0}\left(\lambda-\lambda^{-2}\right)\right]^{-1}$ versus $\lambda^{-1}$. In general these plots are of rather different shape for each of the two polymers; for a given $C_{1}$ value, the deviation from linearity occasioned by the incidence of finite chain extensibility effects occurs at rather greater $\lambda$ values for vulcanizates of cis-1,4-polyisoprene than for those of natural rubber and further, $C_{2}$ values are somewhat larger for the former material. Since $C_{1}$ values are determined by extrapolation of the linear portions of these plots, relatively slight differences in the procedure adopted for this extrapolation between vulcanizates of natural rubber and cis-1,4-polyisoprene could readily account for small systematic differences in the values of $C_{1}$ estimated for the two polymers.

Application of the modified Flory equation, ${ }^{9}$

$$
\begin{equation*}
-\ln \left(1-v_{r}\right)-v_{r}-\chi^{\prime}{ }_{\mathrm{D}} v_{r}^{2}=\left(2 V_{0} C_{1} / R T\right)\left(v_{r}^{1 / 3}-v_{r} / 2\right) \tag{5}
\end{equation*}
$$

gives values of $\chi^{\prime}$ ( (Table I) which are greater than $\chi_{\mathrm{D}}$ and show a greater dependence on $v_{r}$. Inasmuch as there is no independent evidence as to the value of the interaction parameter for the natural rubber- $n$-decane system, or its concentration dependence in this particular $v_{r}$ range, these data in no way constitute an experimental test of the relative merits of eqs. (4) and (5).

For some of the vulcanizate networks, values of $v_{\tau}$ in benzene have been estimated, and $\chi_{\mathrm{B}}$ and $\chi^{\prime}$ в calculated (Table I). For this solvent the extensive work of Gee and his collaborators ${ }^{10}$ has provided values of the solvent interaction parameter of $0.40-0.42$ for $0.1<v_{r}<0.9$. While these values apply only to unvulcanized natural rubber, the crosslinking of rubber chains without any modification or incorporation of foreign atoms would not be expected to induce any significant change in the value of $\chi$, a conclusion for which theoretical evidence has been adduced. ${ }^{11}$ The vulcanization of natural rubber by dicumyl peroxide is believed to be such an uncomplicated crosslinking process, ${ }^{12,13}$ but the possible introduction of some cyclized units into the main chain cannot be entirely discounted. Gee's ${ }^{10}$ value of $0.40-0.42$ is in closer accord with the values estimated for $\chi_{\mathrm{B}}$ from eq. (4) than those of $\chi^{\prime}$ в deduced from eq. (5) and to this extent these present data may be said to favor eq. (4).

The data for $n$-heptane (Table I) also provide evidence from which the same conclusion can be drawn. In this case the value of $\chi^{\prime}{ }_{\mathbf{H}}$ is markedly dependent on $v_{r}$ and for $v_{r}>$ ca. $0.17, \chi^{\prime}{ }_{H}>0.50$. If these values are also applicable to the unvulcanized rubber then, dependent on the molecular
weight, partial miscibility of natural rubber and $n$-heptane should be observed above a critical value of $v_{r}$, the critical conditions being. ${ }^{14}$.

$$
\begin{gather*}
\chi_{\text {crit. }}=0.5+(M / 68)^{-1 / 2}  \tag{6}\\
v_{\text {rerit. }}=(M / 68)^{-1 / 2} \tag{7}
\end{gather*}
$$

where $M$ is the molecular weight of the rubber.
Partial miscibility is not observed with $n$-heptane, and hence either the values of $\chi^{\prime}{ }_{H}$ are not applicable to the unvulcanized state or the values of $\chi_{H}$ from eq. (4) are to be preferred.

The above arguments in favor of the original form of the Flory equation [eq. (4)] in preference to the later modified form [eq. (5)] are clearly only valid insofar as the precision of $C_{1}$ as a measure of the degree of physically effective crosslinking is considerably greater than the difference between the $v_{r}^{1 / 3}$ and ( $v_{r}^{1 / 3}-v_{r} / 2$ ) terms in eqs. (4) and (5). While the significance of $C_{1}$ measurements on unswollen vulcanizates (such as are reported here) might be said to be obscured by the existence of a comparable $C_{2}$ value, the close parity (ca. $5 \%$ ) which exists in favorable cases between $C_{1}$ values obtained for unswollen vulcanizates and those for the same vulcanizates when highly swollen when $C_{2} \rightarrow 0$ supports the contention that $C_{1}$ as measured here is in fact a sufficiently precise measure of physically effective crosslinking.

In conclusion it must be admitted that despite the above evidence adduced in support of the original form of the Flory equation [eq. (4)] in preference to the later modified form [eq. (5)], these equations are based on theoretical treatments which are not entirely satisfactory and both may well be incorrect to an extent greater than the difference between the terms $v_{r}^{1 / 2}$ and ( $v_{r}^{1 / 2}-v_{r} / 2$ ) would imply, since this difference amounts to only some $20 \%$ for $v_{\tau}=0.2-0.3$.

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## Résumé

On décrit le comportement élastique au moyen des paramètres $C_{1}$ et $C_{2}$, et l'équilibre de gonflement dans le $n$-décane, $v_{r}$, pour le caoutchouc naturel et le cis-1,4-polyisoprène vulcanisés au peroxyde de dicumyle. Pour le caoutchouc naturel vulcanisé, la corrélation entre $C_{1}$ et $v_{r}$ peut être décrite au moyen de l'équation de Flory-Rehner, avec $\chi=$ 0.41. On démontre que cette équation est plus favorable que la forme modifée ultèrieurement dans laquelle le terme en $v_{r}^{1 / 3}$ est remplacé par un autre en ( $v_{r}^{1 / 3}-v_{r} / 2$ ).

## Zusammenfassung

Ergebnisse für das elastische, durch die Parameter $C_{1}$ und $C_{2}$ beschriebene Verhalten und die Gleichgewichtsquellung in $n$-Dekan, $v_{r}$, von Dicumylperoxydvulkanisaten von Naturkauschuk und cis-1,4-Polyisopren werden mitgeteilt. Bei Naturkauschukvulkanisaten kann die Beziehung zwischen $C_{1}$ und $v_{r}$ durch die ursprüngliche Flory-RehnerGleichung mit $x=0,41$ wiedergegeben werden. Vorläufige Ergebnisse lassen erkennen, dass diese Gleichung besser zutrifft als die später modifizierte Form, bei welcher der Term mit $v_{r}^{1 / 2}$ durch einen mit ( $v_{r}^{1 / 8}-v_{r} / 2$ ) ersetzt wird.

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